

**FACULTY OF INFORMATICS****B.E. 2/4 (I.T.) II - Semester (Suppl.) Examination, December / January 2014-15****Subject: Probability and Random Process****Time: 3 Hours****Max. Marks: 75****Note: Answer all questions of Part - A and answer any five questions from Part - B.****PART – A (25 Marks)**

- 1 State multiplication theorem for n events. (2)
- 2 State Winner Khinchin relation. (2)
- 3 Show that for any two events A and B, (3)  
 $P(A \cap B) \leq P(A) \leq P(A \cup B) \leq P(A) + P(B)$
- 4 Define probability density function of a r.v. in discrete and continuous cases. (2)
- 5 The pdf of the r.v. X and Y is  $f_{XY}(x, y) = Ae^{-(x+y)}$ ,  $x \geq 0, y \geq 0$ . Find the constant A. (3)
- 6 Write the properties of characteristic function of a r.v. (2)
- 7 What is first order stationery process? (3)
- 8 If X and Y are r.v.s. a and b are constant prove that (3)  
 $V(aX - bY) = a^2V(X) + b^2V(Y) - 2ab \text{ COV}(X, Y)$
- 9 Define while noise. (2)
- 10 State the properties of spectral density function. (3)

**PART – B (50 Marks)**

- 11 (a) State and prove Baye's Theorem. (5)  
 (b) State and prove addition theorem of probability of n events. (5)
- 12 Joint distribution of X and Y is given by  $f_{XY}(x, y) = 4xye^{-(x^2+y^2)}$ ;  $x \geq 0, y \geq 0$  (10)  
 (i) Test whether X and Y are independent  
 (ii) For the above joint distribution, find the conditional density X given  $Y = y$  and conditional density Y given  $X = x$ .
- 13 (a) State the Properties auto correlation. (4)  
 (b) If X(t) is random telegraph signal process with  $E(X(t)) = 0$  and  $R(T) = e^{-2\lambda|T|}$  then find mean and variance of the time average of  $\{x(t)\}$  over  $(-T, T)$ . Is it mean ergodic? (6)
- 14 If  $U(t) = X \cos t + Y \sin t$  and  $V(t) = Y \cos t + X \sin t$  where X and Y are independent r.v.s. such that  $E(X) = 0 = E(Y)$ ;  $E(X^2) = E(Y^2) = 1$ ; show that  $\{U(t)\}$  and  $\{V(t)\}$  are individually stationary in the wide sense (WSS) but they are not jointly W.S.S. (10)

- 15 (a) Give the autocorrelation function for a stationary process is  $R_{xx}(\tau) = 25 + \frac{4}{1+6\tau^2}$   
Find the mean and variance of the process  $X(t)$ . (5)
- (b) Given a stationary random process  $X(t) = 10 \cos(100t + \theta)$  where  $\theta$  is a r.v. with a uniform probability distribution in the interval  $[-\pi, \pi]$ . Show that  $X(t)$  is ergodic in the autocorrelation function. (5)
- 16 The joint random variable (x, Y) probabilities function is given by (10)

Y \ X	1	2	3	4	5	6
0	0	0	$\frac{1}{32}$	$\frac{2}{32}$	$\frac{2}{32}$	$\frac{2}{32}$
1	$\frac{1}{16}$	$\frac{1}{16}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$	$\frac{1}{8}$
2	$\frac{1}{32}$	$\frac{1}{32}$	$\frac{1}{64}$	$\frac{1}{64}$	0	$\frac{2}{64}$

Find

- (i)  $P(X \leq 1)$  (ii)  $P(X \leq 1, Y \leq 3)$   
 (iii)  $P(X \leq 1 / Y \leq 3)$  (iv)  $P(X+Y \leq 4)$  (v)  $P(Y \leq 3 / X \leq 1)$
- 17 A white noise of Gaussian process is zero, and  $S(w) = \frac{N_0}{2}$  applied to a low pass RC filter whose transfer function  $H(f) = \frac{1}{1 + 2\pi fRC}$ . Find the autocorrelation function of the output random process. (10)

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