

## FACULTY OF INFORMATICS

B.E. 2/4 (I.T.) II – Semester (New) (Main) Examination, May / June 2016

Subject: Probability &amp; Random Processes

Time: 3 Hours

Max.Marks: 75

Note: Answer all questions from Part A. Answer any five questions from Part B.

## PART – A (25 Marks)

- 1 If  $A, B$  and  $C$  are any three events such that  
 $P(A) = P(B) = P(C) = \frac{1}{4}$ ,  $P(A \cap B) = P(B \cap C) = 0$  and  $P(A \cap C) = \frac{1}{8}$ .  
 Find the probability that atleast one of the events  $A, B, C$  occurs. 3
- 2 State the generalised form of Bernoulli's theorem on independent trials. 2
- 3 For a random variable  $X$  the probability distribution is given by  
 $P(x) = \frac{k}{2^x}$  for  $x = 0, 1, 2, 3, 4$ . Find the mean and variance of  $X$ . 3
- 4 A random variable  $X$  takes the values 1 and 0 with probabilities  $p$  and  $(1-p)$  respectively. Find the variance of  $X$ . 2
- 5 State the properties of characteristic function. 2
- 6 Write the probability function of two dimensional random variable  $(X, Y)$  3
- 7 If  $X$  and  $Y$  are independent random variables then show that  $E\left(\frac{Y}{X}\right) = E(Y)$  2
- 8 Define stationary process. What are the necessary and sufficient conditions for a process to be stationary? 3
- 9 State Weiner-Khinchine theorem. 2
- 10 Define Guassian process. State its properties. 3

## PART – B (5x10 = 50 Marks)

- 11 a) State and prove Baye's theorem. 3
- b) For a certain binary communication channel, the probability that a transmitted 0 is received as a 0 is 0.95 and the probability that a transmitted 1 is received as 1 is 0.90. If the probability that a 0 is transmitted is 0.4, find the probability that (i) a 1 is received (ii) a 1 was transmitted given that a 1 was received. 7
- 12 a) Define cumulative distribution function of a random variable. State its properties. 4
- b) A random variable  $X$  has the following probability distribution 6

$x$	-2	-1	0	1	2	3
$P(x)$	0.1	$k$	0.2	$2k$	0.3	$3k$

- i) Evaluate  $P(X < 2)$  and  $P(-2 < X < 2)$  (ii) find c.d.f. of  $X$  (iii) evaluate  $E(X)$

...2.

13 For the joint probability distribution of two random variables  $X$  and  $Y$  given below 10

$X \backslash Y$	1	2	3	4
1	4/36	3/36	2/36	1/36
2	1/36	3/36	3/36	2/36
3	5/36	1/36	1/36	1/36
4	1/36	2/36	1/36	5/36

Find

- Marginal distributions of  $X$  and  $Y$
- Conditional distribution of  $X$  given the value of  $Y = 1$
- Conditional distribution of  $Y$  given the value of  $X = 2$

14 a) Define auto-correlation and state its properties. 5

b) Define power spectral density function. State its properties. 5

15 a) Find the mean and auto-correlation of Poisson process. 4

- b) Suppose that customers arrive at a bank according to a Poisson process with mean rate of 3 per minute. Find the probability that during a time interval of 2 minutes
- exactly 4 customers arrive
  - more than 4 customers arrive. 6

16 a) The joint p.d.f. of r.v.  $(X, Y)$  is given by  $f(x, y) = k x y e^{-(x^2 + y^2)} x > 0, y > 0$ . Find the value of  $k$  and prove that  $X$  and  $Y$  are independent. 5

- b) 15% of the employees of a firm are B.E. degree holders, 25% are M.B.A. degree holders and 5% have both the degrees. Find the probability of selecting a B.E. degree holder, if the selection is confined to M.B.A. degree holder. 5

17 a) For a distribution mean is 10, variance is 16,  $\gamma_1 = 1$  and  $\beta_2 = 4$ . Obtain the first four moments about origin. 5

- b) Show that spectral density and the autocorrelation function of a real SSS process form a Fourier Cosine transform pair. 5

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