

## LOGARITHMS

1.  $\log_a (x y) = \log_a x + \log_a y$
2.  $\log_a [x/y] = \log_a x - \log_a y$
3.  $\log_x x = 1$
4.  $\log_a 1 = 0$
5.  $\log_a x^n = n (\log_a x)$
6.  $\log_a x = 1 / \log_x a$
7.  $\log_a x = \log_b x / \log_b a = \log x / \log a$

### Problems with solutions

1. If  $\log 2 = 0.3010$  and  $\log 3 = 0.4771$ , the value of  $\log_5 512$  is:

#### Solution

$$\begin{aligned}\log_5 512 &= \frac{\log 512}{\log 5} \\ &= \frac{\log 2^9}{\log (10/2)} \\ &= \frac{9 \log 2}{\log 10 - \log 2} \\ &= \frac{(9 \times 0.3010)}{1 - 0.3010} \\ &= \frac{2.709}{0.699} \\ &= \frac{2709}{699} \\ &= 3.876\end{aligned}$$

2. If  $\log_{10} 5 + \log_{10} (5x + 1) = \log_{10} (x + 5) + 1$ , then x is equal to:

#### Solution

$$\begin{aligned}\log_{10} 5 + \log_{10} (5x + 1) &= \log_{10} (x + 5) + 1 \\ \log_{10} 5 + \log_{10} (5x + 1) &= \log_{10} (x + 5) + \log_{10} 10 \\ \log_{10} [5 (5x + 1)] &= \log_{10} [10(x + 5)] \\ 5(5x + 1) &= 10(x + 5) \\ 5x + 1 &= 2x + 10\end{aligned}$$

$$3x = 9$$
$$x = 3.$$

3. If  $\log_x\left(\frac{9}{16}\right) = -\frac{1}{2}$ , then x is equal to:

**Solution**

$$\log_x\left(\frac{9}{16}\right) = -\frac{1}{2}$$

$$\Rightarrow x^{-1/2} = \frac{9}{16}$$

$$\Rightarrow \frac{1}{x} = \frac{9}{16}$$

$$\Rightarrow x = \frac{16}{9}$$

$$\Rightarrow x = \left(\frac{16}{9}\right)^2$$

$$\Rightarrow x = \frac{256}{81}$$

4. If  $\log_x y = 100$  and  $\log_2 x = 10$ , then the value of y is:

**Solution**

$$\log_2 x = 10 \Rightarrow x = 2^{10}.$$

$$\log_x y = 100$$

$$\Rightarrow y = x^{100}$$

$$\Rightarrow y = (2^{10})^{100} \quad [\text{put value of } x]$$

$$\Rightarrow y = 2^{1000}.$$

5. If  $\log_{10} 2 = 0.3010$ , then  $\log_2 10$  is equal to:

**Solution**

$$\log_2 10 = \frac{1}{\log_{10} 2} = \frac{1}{0.3010} = \frac{10000}{3010} = \frac{1000}{301}.$$