NUMBERS

S.NO	FORMULAE
1	$(a + b)(a - b) = a^2 - b^2$
2	$(a + b)^2 = a^2 + b^2 + 2ab$
3	$(a - b)^2 = a^2 + b^2 - 2ab$
4	$(a + b + c)\ 2 = a^2 + b^2 + c^2 + 2(ab + bc + ca)$
5	$a^3 + b^3 = (a + b)(a^2 - ab + b^2)$
6	$a^3 - b^3 = (a - b)(a^2 + ab + b^2)$
7	$a^3 + b^3 + c^3 - 3abc = (a + b + c)(a\ 2 + b\ 2 + c\ 2 - ab - bc - ac)$
8	When $a + b + c = 0$, then $a\ 3 + b\ 3 + c\ 3 = 3abc$

Problems with solutions

1. Three times the first of three consecutive odd integers is 3 more than twice the third. The third integer is:

Solution
Let 3 integers be x, x + 2 and x + 4.
$3x = 2(x + 4) + 3 \quad x = 11.$
Third integer = x + 4 = 15.

2. A two-digit number is such that the product of the digits is 8. When 18 is added to the number, then the digits are reversed. The number is:

Solution

Let ten's and unit digit be x and $\frac{8}{x}$ Respectively.

Then, $\left(10x + \frac{8}{x}\right) + 18 = 10 \text{ x } \frac{8}{x} + x$

$10x^2 + 8 + 18x = 80 + x^2$
$9x^2 + 18x - 72 = 0$
$x^2 + 2x - 8 = 0$
$(x + 4)(x - 2) = 0$
$x = 2.$

3. The sum of the squares of three numbers is 138, while the sum of their products taken two at a time is 131. Their sum is:

Solution
Let the numbers be a, b and c.
Then, $a^2 + b^2 + c^2 = 138$ and $(ab + bc + ca) = 131$.
$(a + b + c)^2 = a^2 + b^2 + c^2 + 2(ab + bc + ca) = 138 + 2 \text{ x } 131 = 400.$
$(a + b + c) = 400 = 20.$

4. In a two-digit, if it is known that its unit's digit exceeds its ten's digit by 2 and that the product of the given number and the sum of its digits is equal to 144, then the number is:

Solution
Let the ten's digit be x.
Then, unit's digit = x + 2.
Number = 10x + (x + 2) = 11x + 2.
Sum of digits = x + (x + 2) = 2x + 2.
$(11x + 2)(2x + 2) = 144$
$22x^2 + 26x - 140 = 0$
$11x^2 + 13x - 70 = 0$
$(x - 2)(11x + 35) = 0$
$x = 2$.
Hence, required number = 11x + 2 = 24.

5. Find a positive number which when increased by 17 is equal to 60 times the reciprocal of the number.

Solution
Let the number be x.
$x + 17 = \frac{60}{x}$
$x^2 + 17x - 60 = 0$
$(x + 20)(x - 3) = 0$
$x = 3$.