**Information Technology Department**

**IMPORTANT QUESTIONS**

**Probability & Random Processes**

**SHORT ANSWER QUESTIONS**

1. Write axiomatic definition of probability.
2. If A, B & C are any three events such that P(A) = P(B)=P(C)= 1/4, P(A∩B)=P(B∩C)=0 and P(A∩C)= 1/8. Find the probability that at least one of the events A, B, C occurs.
3. State the properties of characteristic function of a random variable.
4. If a random variable X is uniformly distributed over (0, 10), Find its mean & variance.
5. Define Covariance.
6. If X, Y are random variables and a, b are constants then prove that Var (aX+bY) = a2Var(X) + b2Var(Y) + 2ab.Cov(X, Y).
7. Define Mean Ergodic & Correlation Ergodic process.
8. Write any three properties of Autocorrelation.
9. State Wiener-Kintchine theorem.
10. Define White Noise and Colored Noise.
11. State the generalized form of Bernoulli theorem.
12. If A & B are independent events, Show that Ac & Bc are also independent events.
13. Define Cumulative Distribution Function (CDF) of a random variable X. State its properties.
14. A coin is tossed 1000 times, Find the probability of getting 520 heads.
15. Show that Covariance of two independent random variables is 0.
16. If X, Y are random variables and a, b are constants then prove that Var (aX - bY) = a2Var(X) + b2Var(Y) - 2ab.Cov(X, Y).
17. State Ergodicity and Stationarity of a random process.
18. Autocorrelation of a stationary process X(t) is given by Rxx(τ) = 25 + 4/(1+6τ2). Find its mean and variance.
19. Define Gaussian process.
20. Write any three properties of Power Spectral density of a stationary process.
21. If random variable x takes the values 1 and 0 with probability p and q = 1-p. Show that the variance is equal to pq.
22. Explain Joint Moments of a Random Variable.
23. State Addition theorem for n events.
24. What is Joint Characteristic Functions and what is its use.
25. In an experiment of drawing a card from a pack of playing cards, Let A and B be the events of getting a red card and a queen respectively. Show that these events A and B are independent.

**LONG ANSWER QUESTIONS**

1. Two persons A and B alternately throw a pair of die. A wins, if he gets the sum of two dice 7 before B gets 9. B wins, if he gets the sum of two dice 9 before A gets 7. If A starts the game, find the probability that A wins the game.

2. A box B1 contains 10 white and 3 black balls, and another box B2 contains 3 white and 5 black balls. Two balls are selected at random from the first box B1 and placed in the second box B2. Then a ball is taken out at random from the second box B2.

i) Find the probability that it is a white ball.

ii) If it is a white ball find the probability that 2 black balls are moved from B1 to B2.

3. In a game of rolling a pair of dice a random variable X is defined as sum of the numbers on the dice. Find the Moment Generating Function of the random variable X, & hence compute its mean & variance.

4. If a continuous RV ‘X’ has a pdf fx(x) = { ; -1 < x < 2. Find the pdf of Y such that Y=X2.

1. Find the following, if joint density of two continuous random variables X & Y is given by

f(x, y) = k.(x3y +xy3) ; 0 < x <2 & 0 <y< 2

i) k ii) Marginal density functions of X & Y iii) Cov(X, Y)

iv) E[ X/Y ] v) E[ Y/X ]

1. Define stationary process. What are the necessary and sufficient conditions for a process to be stationary?
2. If X(t) = 5cos(10t + θ) & Y(t) = 20sin(10t + θ ) where θ is a uniformly distributed random variable in (0, 2π). Prove that X(t) & Y(t) are jointly stationary in wide sense WSS.
3. Consider a white Gaussian noise of zero mean and power spectral density No/2 applied to a low-pass RC filter whose transfer function is given below. Find the auto-correlation function of the output random process.

H(f) =

1. A person X speaks truth 4 out of 5 times. A die is rolled & he reports that it is a 6. Find the probability that it was actually a 6.

1. Derive the expressions for mean & variance of exponential random variable.
2. For the joint probability distribution of two discrete random variables X& Y is given below. Find

i) Marginal distributions of X& Y

ii) Conditional Mean of X given the value of Y=1

|  |  |  |  |
| --- | --- | --- | --- |
| X**\**Y | 1 | 2 | 3 |
| 1 | 4/36 | 3/36 | 2/36 |
| 2 | 3/36 | 7/36 | 5/36 |
| 3 | 5/36 | 2/36 | 5/36 |

1. Over a period of 12 hours, 180 calls are made at random. What is the probability that in a two hour interval the number of calls is between 25 and 35?
2. In a game of rolling a pair of dice, a player wins the game if he gets the sum 7 or 11 &losses the game if he gets the sum 2, 3 or 12. Any other sum is called a “carry over”. If outcome of a throw is a carry over, the player throws the dice again until he wins or loses the game. The player also loses the game if he gets seven carry overs. What is the probability that the player wins the game?
3. State and prove Bayes theorem.
4. The Probability Density Function (pdf) of a continuous random variable X that can take values between X=2 and X=5 is given by f(x) = k.(1+x) Find

i) k ii) Mean iii) Variance iv) P(X<4)

1. Find Moment generating function of Binomial random variable, and hence find its mean.
2. Let X & Y are two continuous random variables with joint density function f(x, y) = 4xy ; 0 < x < 1 & 0 < y < 1 and f(x, y) = 0 ; elsewhere. Find

i) Var (X) ii) Var (Y) iii)Cov(X, Y)

iv) P(X< 1/2) v) P (Y>1/4) vi) P(X< 1/2,Y>1/4)

1. Define stationary process. What are the necessary and sufficient conditions for a process to be stationary?
2. If U(t) = X.cost + Y.sint & V(t) = Y.cost + X.sint where X, Y are independent random variables such that E[X]=E[Y]=0, E[X2]=E[Y2]=1. Show that U(t) &V(t) are individually stationary in the wide sense (WSS), but they are not jointly stationary in the wide sense (WSS)
3. Consider a white Gaussian noise of zero mean and power spectral density No/2 applied to a low-pass RL filter whose transfer function is given below. Find the auto-correlation function of the output random process.

H(f) =

1. A speaks truth in 70% of cases and B speaks truth in 80% of cases. Find the probability that they contradict each other while speaking the same incident.
2. A discrete random variable has the following probability distribution.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x | 1 | 2 | 3 | 4 | 5 | 6 |
| P(X= x) | k | 3k | 5k | 7k | 9k | 11k |

Find i) k ii)Mean iii) Variance iv) P(1<X<5)

1. If X, Y are two independent exponential random variables with common parameter 1. Find joint and marginal pdf ‘s of U and V such that U=X+Y & V=X-Y
2. For a random process having Autocorrelation Rxx(τ) = a. e -b| τ|. Find Power spectral density.
3. Suppose box 1 contains a white balls and b black balls and box 2 contain c white balls and d black balls. One ball of unknown color is transferred from the first box into the second one and then a ball is drawn from the latter. What is the probability that it will be a white ball?
4. If a continuous RV ‘X’ has a pdf fx(x) = 2(x-1) ; 1 < x < 2.

Find the pdf of Y such that Y=X2 .

1. Find the following, if joint density of two continuous random variables X & Y is given by

f(x, y) = k.(x2 +y2) ; 0 < x <2 & 0 <y< 2

i) k ii) Marginal density function of X

iii) Marginal density function of Y iv) Cov(X, Y)

1. If X(t) = 5cos(10t + θ) where θ is a uniformly distributed random variable in (0, 2π). Prove that X(t) is stationary in wide sense WSS.

1. Derive the expressions for mean & variance of binomial random variable.
2. For the joint probability distribution of two discrete random variables X & Y is given below. Find

i) Marginal distributions of X

ii) Conditional Mean of X given the value of Y=1

iii) P(X<2, Y<3)

iv) P(X+Y<4)

v) P(X<2/Y<3)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| X**\**Y | 1 | 2 | 3 | 4 | 5 |
| 1 | 1/36 | 3/36 | 2/36 | 1/36 | 3/36 |
| 2 | 3/36 | 2/36 | 3/36 | 2/36 | 2/36 |
| 3 | 4/36 | 2/36 | 3/36 | 4/36 | 1/36 |

1. Two players A and B draw balls one at a time alternatively from a box containing m white balls and n black balls. Suppose the player who picks the first white ball wins the game, what is the probability that the player who starts the game will win?
2. a. The Probability Density Function (pdf) of a continuous random variable X that can take values between X=2 and X=5 is given by f(x) = k(1+x2). Find

i) k ii) Mean iii) Variance iv) P(X<4)

v) P(X>3) vi) P(3<X<4)

1. Let X(t) = A cos wt + B sin wt, Y(t) = B cos wt – A sin wt where A and B are random variables, w is a constant, show that X(t) and Y(t) are wide sense stationary if A and B are uncorrelated, with zero mean and same variance.
2. State and prove Wiener-Kintchine theorem.
3. Explain conditional probability.
4. If X, Y are two independent exponential random variables with common parameter 1. Find density function of U such that U=X+Y .

37. A lot contains of 10 good articles, 4 with minor defects & 2 with major defects. If two articles are chosen from the lot at random, Find the probability that

i) Both are good ii) Neither is good iii) At least one is good iv) At most one is good

v) Exactly one is good vi) Both have major defects vii) Neither has major defects

38. State & prove generalized form of Bernoulli theorem. Give an example to demonstrate its application.

39. Show that the area under the Normal curve is unity.

40. Derive an expression for nth moment (Mn) of a random variable X in terms of its characteristic function.

41. Two random variables X & Y have joint density function f(x, y) = 8xy ; 0 < x < y< 1. Find

i) Marginal density functions of X & Y ii) Var (X) iii) Var (Y)

iv) Cov(X, Y) v) P(X< 1/2, Y<1/4 )

42. Explain the properties of Auto-correlation & Cross-correlation.

43. If X(t) = A.cos(wt + θ) and Y(t) = B.sin(wt + θ ), where A, B, w are constants and θ is a uniformly distributed random variable in (0, 2π). Verify whether X(t) and Y(t) are jointly stationary in wide sense or not.

44. Suppose that customers arrive at a bank according to a Poisson process with mean rate of 3 per minute. Find the probability that during a time interval of 2 minutes.

i) exactly 4 customers arrive (ii) more than 4 customers

45. Machines A & B produce 30% & 70% of a factory’s output. Machine A produces 3% defectives & Machine B produces 4% defectives. If an item is selected at random, & found to be

i) Defective, What is the probability that it was produced by Machine A.

ii) Non defective, what is the probability that it was produced by Machine A

46. Determine the following if f (x, y) = {k ; 0 xy1

 {0; otherwise

i) f (x/y) ii) f (y/x) iii) E [y/x] iv) Var [y/x]

47. For the joint probability distribution of two discrete random variables X& Y is given below. Find

i) Marginal distributions of X& Y

ii) Conditional Mean of X given the value of Y=1

iii) Conditional Mean of Y given the value of X=2

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
| X**\**Y | 1 | 2 | 3 | 4 |
| 1 | 4/36 | 3/36 | 2/36 | 1/36 |
| 2 | 1/36 | 3/36 | 3/36 | 2/36 |
| 3 | 5/36 | 1/36 | 1/36 | 1/36 |
| 4 | 1/36 | 2/36 | 1/36 | 5/36 |

48. Define the semi-random telegraph signal process & random telegraph signal process. And also prove that the former is Evolutionary & the latter is wide sense stationary process.

50. Find auto-correlation function of a WSS process if its power spectral density (PSD) is given by

SXX(w) = { b / a (a - | w |); |w | <= a

{ 0; | w | > a

51. Two dice are rolled 7 times. If getting a doublet is a success. Find the probability of getting

i) At least 5 successes ii) At most 2 successes

1. For a Poisson random variable P(X=2) = 2.P(X=0), Find

i) Mean ii) P(2< X <6) iii) P(X>2) `

1. If X(t) = 5cos(10t + θ) and Y(t) = 20sin(10t + θ ) where θ is a uniformly distributed random variable in (0, 2π). Prove that X(t) and Y(t) are jointly stationary in wide sense WSS.