**Information Technology Department**

**IMPORTANT QUESTIONS OF PROBABILITY & RANDOM PROCESSES**

**SHORT ANSWER QUESTIONS**

**Unit-1**

1. Write axiomatic definition of probability.
2. If A, B & C are any three events such that P(A) = P(B)=P(C)= 1/4, P(A∩B)=P(B∩C)=0 and P(A∩C)= 1/8. Find the probability that at least one of the events A, B, C occurs.
3. If A & B are independent events, Show that Ac & Bc are also independent events.
4. In an experiment of drawing a card from a pack of playing cards, Let A and B be the events of getting a red card and a queen respectively. Show that these events A and B are independent.
5. A box contains 4 white, 5 red & 6 black balls. Four balls are drawn at random from the box. Find the probability of drawing at least one ball of each color.
6. There are two bags one of which contains 5 red & 8 black balls and the other bag contains 7 red & 10 black balls. A ball is drawn from one or the other of the two bags. Find the probability of drawing a red ball.
7. State Addition theorem for n events.
8. State the generalized form of Bernoulli theorem.

**Unit-2**

1. Define Cumulative Distribution Function (CDF) of a random variable X. State its properties.
2. If random variable x takes the values 1 and 0 with probability p and q = 1-p. Show that the variance is equal to pq.
3. If a random variable X is uniformly distributed over (0, 10), Find its mean & variance.
4. The amount of waiting time a customer spends at a restaurant has exponential distribution with a mean value of 5 minutes. Find the probability that the customer will spend

i) More than 10 minutes in the restaurant.

ii) Additional 10 minutes in the restaurant given that he has been there for more than 10 minutes.

1. The probability that a driver will have an accident in one month equals 0.02. Find the probability that in 100 months he will have i) at least two accidents ii) three accidents.
2. A coin is tossed 1000 times. Find the probability of getting 520 heads.
3. A fair coin is tossed 900 times. Find the probability that the number of heads is between 420 & 465.
4. State the properties of characteristic function of a random variable.

**Unit-3**

1. Write any four properties of joint probability density function of two random variables.
2. Define Covariance.
3. Show that Covariance of two independent random variables is 0.
4. If X, Y are random variables and a, b are constants then prove that Var (aX+bY) = a2Var(X) + b2Var(Y) + 2ab.Cov(X, Y).
5. Explain Joint Moments of Random Variables.
6. What is Joint Characteristic Functions and what is its use.

**Unit-4**

1. Define stationary process.
2. What are the necessary and sufficient conditions for a process to be stationary?
3. Define Mean Ergodic & Correlation Ergodic process.
4. Write any three properties of Autocorrelation.
5. Autocorrelation of a stationary process X(t) is given by Rxx(τ) = 25 + 4/(1+6τ2). Find its mean and variance.
6. Find second moment of a random variable X(8) – X(5), where X(t) is a WSS process with auto-correlation Rxx(τ) = a. e -b| τ|.

**Unit-5**

1. Write any three properties of Power Spectral density of a stationary process.
2. State Wiener-Kintchine theorem.
3. Define Gaussian process.
4. Define White Noise and Colored Noise.

**LONG ANSWER QUESTIONS**

**Unit-1**

1. A lot contains of 10 good articles, 4 with minor defects & 2 with major defects. If two articles are chosen from the lot at random, Find the probability that

i) Both are good ii) Neither is good iii) At least one is good iv) At most one is good

v) Exactly one is good vi) Both have major defects vii) Neither has major defects

1. Explain conditional probability.
2. Two persons A and B alternately throw a pair of die. A wins, if he gets the sum of two dice 7 before B gets 9. B wins, if he gets the sum of two dice 9 before A gets 7. If A starts the game, find the probability that A wins the game.
3. A speaks truth in 70% of cases and B speaks truth in 80% of cases. Find the probability that they contradict each other while speaking the same incident.
4. In a game of rolling a pair of dice, a player wins the game if he gets the sum 7 or 11 &losses the game if he gets the sum 2, 3 or 12. Any other sum is called a “carry over”. If outcome of a throw is a carry over, the player throws the dice again until he wins or loses the game. The player also loses the game if he gets seven carry overs. What is the probability that the player wins the game?
5. Two players A and B draw balls one at a time alternatively from a box containing m white balls and n black balls. Suppose the player who picks the first white ball wins the game, what is the probability that the player who starts the game will win?
6. State and prove Bayes theorem.

8. A box B1 contains 10 white and 3 black balls, and another box B2 contains 3 white and 5 black balls. Two balls are selected at random from the first box B1 and placed in the second box B2. Then a ball is taken out at random from the second box B2.

i) Find the probability that it is a white ball.

ii) If it is a white ball find the probability that 2 black balls are moved from B1 to B2.

1. For a binary communication channel the probability that a transmitted 0, received as 0 is 0.95. The probability that a transmitted 1, received as 1 is 0.9. If the probability that a 0 is transmitted is 0.4, Find the probability that
2. 1 is received
3. If 1 is received , What is the probability that 1 was transmitted.
4. A person X speaks truth 4 out of 5 times. A die is rolled & he reports that it is a 6. Find the probability that it was actually a 6.
5. Suppose box 1 contains a white balls and b black balls and box 2 contain c white balls and d black balls. One ball of unknown color is transferred from the first box into the second one and then a ball is drawn from the latter. What is the probability that it will be a white ball?

12. Machines A & B produce 30% & 70% of a factory’s output. Machine A produces 3% defectives & Machine B produces 4% defectives. If an item is selected at random, & found to be

i) Defective, What is the probability that it was produced by Machine A.

ii) Non defective, what is the probability that it was produced by Machine A

13. A pair of dice is rolled 50 times. Find the probability of obtaining double six

i) at least three times ii) exact four times

1. State & prove generalized form of Bernoulli theorem. Give an example to demonstrate its application.

**Unit-2**

1. A discrete random variable has the following probability distribution.

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
| x | 1 | 2 | 3 | 4 | 5 | 6 |
| P(X= x) | k | 3k | 5k | 7k | 9k | 11k |

Find i) k ii)Mean iii) Variance iv) P(1<X<5)

1. The Probability Density Function (pdf) of a continuous random variable X that can take values between X=2 and X=5 is given by f(x) = k.(1+x) Find

i) k ii) Mean iii) Variance iv) P(X<4)

1. In a continuous distribution the probability density is given by f(x) = kx(2-x), 0<x<2. Find i) k ii) Mean iii) Variance iv) CDF
2. Derive the expressions for mean & variance of exponential random variable.
3. Show that the area under the Normal curve is unity.
4. A passenger arrives at a bus stop at 10 A.M., knowing that the bus will arrive at some time uniformly distributed between 10A.M. to 10:30 A.M.

i) Find the probability that he will have to wait for more than 10 minutes.

ii) If he arrives at 10:15 A.M. find the probability that he will have to wait for at least 10 minutes.

1. Derive the expressions for mean & variance of binomial random variable
2. For a Poisson random variable P(X=2) = 2.P(X=0), Find

i) Mean ii) P(2< X <6) iii) P(X>2)

1. Over a period of 12 hours, 180 calls are made at random. What is the probability that in a two hour interval the number of calls is between 25 and 35?
2. In a game of rolling a pair of dice a random variable X is defined as sum of the numbers on the dice. Find the Moment Generating Function of the random variable X, & hence compute its mean & variance.
3. Find Moment generating function of Binomial random variable, and hence find its mean.
4. If a continuous RV ‘X’ has a pdf fx(x) = { ; -1 < x < 2. Find the pdf of Y such that Y=X2.
5. If a continuous RV ‘X’ has a pdf fx(x) = 2x ; 0 < x < 1. Find the pdf of Y such that Y=4X2.
6. Derive an expression for nth moment (Mn) of a random variable X in terms of its characteristic function. **Unit-3**
7. For the joint probability distribution of two discrete random variables X& Y is given below. Find

i) Marginal distributions of X& Y

ii) Conditional Mean of X given the value of Y=1

|  |  |  |  |
| --- | --- | --- | --- |
| X**\**Y | 1 | 2 | 3 |
| 1 | 4/36 | 3/36 | 2/36 |
| 2 | 3/36 | 7/36 | 5/36 |
| 3 | 5/36 | 2/36 | 5/36 |

1. For the joint probability distribution of two discrete random variables X & Y is given below. Find

i) P(X<2, Y<3)

ii) P(X+Y<4)

iii) P(X<2/Y<3)

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| X**\**Y | 1 | 2 | 3 | 4 | 5 |
| 1 | 1/36 | 3/36 | 2/36 | 1/36 | 3/36 |
| 2 | 3/36 | 2/36 | 3/36 | 2/36 | 2/36 |
| 3 | 4/36 | 2/36 | 3/36 | 4/36 | 1/36 |

1. Find the following, if joint density of two continuous random variables X & Y is given by

f(x, y) = k.(x3y +xy3) ; 0 < x <2 & 0 <y< 2

i) k ii) Marginal density functions of X & Y iii) Cov(X, Y)

32. Two random variables X & Y have joint density function f(x, y) = 8xy ; 0 < x < y< 1. Find

i) Marginal density functions of X & Y ii) Var (X) iii) Var (Y)

iv) Cov(X, Y) v) P(X< 1/2, Y<1/4 )

1. Let X & Y are two continuous random variables with joint density function f(x, y) = 4xy ; 0 < x < 1 & 0 < y < 1. Find

i) Var (X) ii) Var (Y) iii)Cov(X, Y)

iv) P(X< 1/2) v) P (Y>1/4) vi) P(X< 1/2,Y>1/4)

1. Two random variables X & Y are jointly distributed over the region 0 x y 1 as

 f (x, y) = {kx ; 0 x y 1

{0 ; otherwise

1. Find k ii) Means of X & Y iii) Variances of X & Y iv) Co-variance of X & Y v) Coefficient of correlation of X & Y
2. If X & Y are two independent exponential random variables with common parameter 3. Two functions U and V are defined, such that U=X+Y & V=X-Y. Find the joint and marginal probability density functions of U and V.
3. Two independent random variables X & Y each following uniform distribution U(0, 1). Two variables Z & W are defined as Z=X+Y & W=X-Y. Find joint density function & marginal density functions of Z & W. Show that Z & W are not independent but uncorrelated random variables.
4. If X, Y are two independent exponential random variables with common parameter 1. Find density function of U such that U=X+Y .
5. Two independent random variables X & Y have probability density functions f(x) = 1; 1 < x < 2, & f(y) = y/6; 2 < y< 4. Find the density function of Z=X.Y
6. Two random variables X & Y have joint probability density function f(x, y) = 1 ; 0 <|y|<x< 1. Find

i) E[X/Y] ii) E[Y/X]

40. Determine the following if f (x, y) = {k ; 0 xy1

 {0; otherwise

i) f (x/y) ii) f (y/x) iii) E [Y/X] iv) Var [Y/X]

**Unit-4**

1. If X(t) = 5cos(10t + θ) where θ is a uniformly distributed random variable in (0, 2π). Prove that X(t) is stationary in wide sense WSS.
2. If X(t) = A.cos(wt + θ) and Y(t) = B.sin(wt + θ ), where A, B, w are constants and θ is a uniformly distributed random variable in (0, 2π). Verify whether X(t) and Y(t) are jointly stationary in wide sense or not.
3. If X(t) = 5cos(10t + θ) & Y(t) = 20sin(10t + θ ) where θ is a uniformly distributed random variable in (0, 2π). Prove that X(t) & Y(t) are jointly stationary in wide sense WSS.
4. If U(t) = X.cost + Y.sint & V(t) = Y.cost + X.sint where X, Y are independent random variables such that E[X]=E[Y]=0, E[X2]=E[Y2]=1. Show that U(t) &V(t) are individually stationary in the wide sense (WSS), but they are not jointly stationary in the wide sense (WSS)
5. Let X(t) = A cos wt + B sin wt, Y(t) = B cos wt – A sin wt where A and B are random variables, w is a constant, show that X(t) and Y(t) are wide sense stationary only if A and B are uncorrelated random variables, with zero mean and same variance.
6. Show that the process X(t) = 10cos(5t + θ),where θ is a uniformly distributed over (0, 2π) is

i) Mean Ergodic ii) Auto-correlation Ergodic

1. Explain the properties of Auto-correlation & Cross-correlation.
2. Find the Auto-correlation of a process X(t)= Y(t) cos(wt+ θ) where Y(t) is a WSS process with auto-correlation Ryy(τ) = e -a| τ |, w is a constant & θ is a uniform random variable over (-π, π), independent to Y(t).
3. A random process is defined as X(t) = B cos(wt), where w is a constant & B is a uniform random variable over (0, 1). Find the Auto-correlation & Auto-covariance of X(t).

**Unit-5**

1. State and prove Wiener-Kintchine theorem.
2. Explain the properties of Power spectral density.
3. For a random process having Autocorrelation Rxx(τ) = a. e -b| τ|. Find Power spectral density.
4. Find Power spectral density of a WSS process with auto-correlation Rxx(τ) = a. exp(-bτ2).
5. Find Power spectral density (PSD) of a random binary transmission process whose auto-correlation function (ACF) is

RXX (τ) = {1- ; |τ | < T

{0 ; else where

1. Find auto-correlation function of a WSS process if its power spectral density (PSD) is given by

SXX(w) = { b / a (a - | w |) ; |w | <= a

{ 0 ; | w | > a

1. Consider a white Gaussian noise of zero mean and power spectral density No/2 applied to a low-pass RC filter whose transfer function is given below. Find the auto-correlation function of the output random process.

H(f) =

57. Consider a white Gaussian noise of zero mean and power spectral density No/2 applied to a low-pass RL filter whose transfer function is given below. Find the auto-correlation function of the output random process.

H(f) =

58. Suppose that customers arrive at a bank according to a Poisson process with mean rate of 3 per minute. Find the probability that during a time interval of 2 minutes.

i) exactly 4 customers arrive (ii) more than 4 customers

59. If particles are emitted from a radioactive source at the rate of 20 per hour according to Poisson process. Find the probability that in a 15 minute interval, the number of emitted particles is

i) Exactly five ii) At least two iii) At most three

60. A Gaussian process X(t) is WSS with mean E[X(t)]=0 & auto-correlation Rxx(τ) = 4 e -3| τ |. Determine i) P(X(t) < 3) ii) E[(X(t+1) - X(t-1))2]