Q. 1 – Q. 5 carry one mark each.

Q.1 The fishermen, _______ the flood victims owed their lives, were rewarded by the government.

(A) whom (B) to which (C) to whom (D) that

Q.2 Some students were not involved in the strike.

If the above statement is true, which of the following conclusions is/are logically necessary?

1. Some who were involved in the strike were students.
2. No student was involved in the strike.
3. At least one student was involved in the strike.
4. Some who were not involved in the strike were students.

(A) 1 and 2 (B) 3 (C) 4 (D) 2 and 3

Q.3 The radius as well as the height of a circular cone increases by 10%. The percentage increase in its volume is ______.

(A) 17.1 (B) 21.0 (C) 33.1 (D) 72.8

Q.4 Five numbers 10, 7, 5, 4 and 2 are to be arranged in a sequence from left to right following the directions given below:
1. No two odd or even numbers are next to each other.
2. The second number from the left is exactly half of the left-most number.
3. The middle number is exactly twice the right-most number.

Which is the second number from the right?

(A) 2 (B) 4 (C) 7 (D) 10

Q.5 Until Iran came along, India had never been ________________ in kabaddi.

(A) defeated (B) defeating (C) defeat (D) defeatist

Q. 6 – Q. 10 carry two marks each.

Q.6 Since the last one year, after a 125 basis point reduction in repo rate by the Reserve Bank of India, banking institutions have been making a demand to reduce interest rates on small saving schemes. Finally, the government announced yesterday a reduction in interest rates on small saving schemes to bring them on par with fixed deposit interest rates.

Which one of the following statements can be inferred from the given passage?

(A) Whenever the Reserve Bank of India reduces the repo rate, the interest rates on small saving schemes are also reduced

(B) Interest rates on small saving schemes are always maintained on par with fixed deposit interest rates

(C) The government sometimes takes into consideration the demands of banking institutions before reducing the interest rates on small saving schemes

(D) A reduction in interest rates on small saving schemes follow only after a reduction in repo rate by the Reserve Bank of India

Q.7 In a country of 1400 million population, 70% own mobile phones. Among the mobile phone owners, only 294 million access the Internet. Among these Internet users, only half buy goods from e-commerce portals. What is the percentage of these buyers in the country?

(A) 10.50 (B) 14.70 (C) 15.00 (D) 50.00

Q.8 The nomenclature of Hindustani music has changed over the centuries. Since the medieval period *dhrupad* styles were identified as *baanis*. Terms like *gayaki* and *baaj* were used to refer to vocal and instrumental styles, respectively. With the institutionalization of music education the term *gharana* became acceptable. *Gharana* originally referred to hereditary musicians from a particular lineage, including disciples and grand disciples.

Which one of the following pairings is NOT correct?

(A) *dhrupad*, *baani*
(B) *gayaki*, vocal
(C) *baaj*, institution
(D) *gharana*, lineage

Q.9 Two trains started at 7AM from the same point. The first train travelled north at a speed of 80km/h and the second train travelled south at a speed of 100 km/h. The time at which they were 540 km apart is ______ AM.

(A) 9 (B) 10 (C) 11 (D) 11.30

Q.10 "I read somewhere that in ancient times the prestige of a kingdom depended upon the number of taxes that it was able to levy on its people. It was very much like the prestige of a head-hunter in his own community."

Based on the paragraph above, the prestige of a head-hunter depended upon ___________

(A) the prestige of the kingdom
(B) the prestige of the heads
(C) the number of taxes he could levy
(D) the number of heads he could gather

END OF THE QUESTION PAPER

Q. 1 – Q. 25 carry one mark each.

Q.1 For a balanced transportation problem with three sources and three destinations where costs, availabilities and demands are all finite and positive, which one of the following statements is **FALSE**?

(A) The transportation problem does not have unbounded solution

(B) The number of non-basic variables of the transportation problem is 4

(C) The dual variables of the transportation problem are unrestricted in sign

(D) The transportation problem has at most 5 basic feasible solutions

Q.2 Let $f:[a,b]\to\mathbb{R}$ (the set of all real numbers) be any function which is twice differentiable in (a,b) with only one root α in (a,b). Let $f'(x)$ and $f''(x)$ denote the first and second order derivatives of $f(x)$ with respect to x. If α is a simple root and is computed by the Newton-Raphson method, then the method converges if

(A) $|f(x)\,f''(x)|<|f'(x)|^2$, for all $x\in(a,b)$

(B) $|f(x)\,f'(x)|<|f''(x)|$, for all $x\in(a,b)$

(C) $|f'(x)\,f''(x)|<|f(x)|^2$, for all $x\in(a,b)$

(D) $|f(x)\,f''(x)|<|f'(x)|$, for all $x\in(a,b)$

Q.3 Let $f:\mathbb{C}\to\mathbb{C}$ (the set of all complex numbers) be defined by

$$f(x+iy)=x^3+3xy^2+i\left(y^3+3x^2y\right),\quad i=\sqrt{-1}.$$

Let $f'(z)$ denote the derivative of f with respect to z.

Then which one of the following statements is TRUE?

(A) $f'(1+i)$ exists and $|f'(1+i)|=3\sqrt{5}$

(B) f is analytic at the origin

(C) f is not differentiable at i

(D) f is differentiable at 1

Q.4 The partial differential equation

$$\left(x^2+y^2-1\right)\frac{\partial^2 u}{\partial x^2}+2\frac{\partial^2 u}{\partial x\partial y}+\left(x^2+y^2-1\right)\frac{\partial^2 u}{\partial y^2}=0$$

is

(A) parabolic in the region $x^2+y^2>2$

(B) hyperbolic in the region $x^2+y^2>2$

(C) elliptic in the region $0<x^2+y^2<2$

(D) hyperbolic in the region $0<x^2+y^2<2$

Q.5 If

$$u_n = \int_1^n e^{-t^2}\, dt, \quad n = 1, 2, 3, \ldots,$$

then which one of the following statements is TRUE?

(A) Both the sequence $\{u_n\}_{n=1}^{\infty}$ and the series $\sum_{n=1}^{\infty} u_n$ are convergent

(B) Both the sequence $\{u_n\}_{n=1}^{\infty}$ and the series $\sum_{n=1}^{\infty} u_n$ are divergent

(C) The sequence $\{u_n\}_{n=1}^{\infty}$ is convergent but the series $\sum_{n=1}^{\infty} u_n$ is divergent

(D) $\lim_{n\to\infty} u_n = \frac{2}{e}$

Q.6 Let $\Gamma = \{(x, y, z) \in \mathbb{R}^3 : -1 < x < 1, -1 < y < 1, -1 < z < 1\}$ and $\phi : \Gamma \to \mathbb{R}$ be a function whose all second order partial derivatives exist and are continuous. If ϕ satisfies the Laplace equation $\nabla^2 \phi = 0$ for all $(x, y, z) \in \Gamma$, then which one of the following statements is TRUE in Γ?

($\mathbb{R}$ is the set of all real numbers, and $\mathbb{R}^3 = \{(x, y, z) : x, y, z \in \mathbb{R}\}$)

(A) $\vec{\nabla}\phi$ is solenoidal but not irrotational

(B) $\vec{\nabla}\phi$ is irrotational but not solenoidal

(C) $\vec{\nabla}\phi$ is both solenoidal and irrotational

(D) $\vec{\nabla}\phi$ is neither solenoidal nor irrotational

Q.7 Let $X=\{(x_1,x_2,\ldots):\ x_i\in\mathbb{R}$ and only finitely many x_i's are non-zero$\}$ and $d:X\times X\to\mathbb{R}$ be a metric on X defined by

$$d(x,y)=\sup_{i\in\mathbb{N}}|x_i-y_i| \text{ for } x=(x_1,x_2,\ldots),\, y=(y_1,y_2,\ldots) \text{ in } X.$$

($\mathbb{R}$ is the set of all real numbers and $\mathbb{N}$ is the set of all natural numbers)

Consider the following statements:

$P:(X,d)$ is a complete metric space.

$Q:$ The set $\{x\in X: d(\underline{0},x)\le 1\}$ is compact, where $\underline{0}$ is the zero element of X.

Which of the above statements is/are TRUE?

(A) Both P and Q (B) P only (C) Q only (D) Neither P nor Q

Q.8 Consider the following statements:

I. The set $\mathbb{Q}\times\mathbb{Z}$ is uncountable.

II. The set $\{f : f \text{ is a function from } \mathbb{N} \text{ to } \{0,1\}\}$ is uncountable.

III. The set $\{\sqrt{p} : p \text{ is a prime number}\}$ is uncountable.

IV. For any infinite set, there exists a bijection from the set to one of its proper subsets.

($\mathbb{Q}$ is the set of all rational numbers, $\mathbb{Z}$ is the set of all integers and $\mathbb{N}$ is the set of all natural numbers)

Which of the above statements are TRUE?

(A) I and IV only (B) II and IV only (C) II and III only (D) I, II and IV only

Q.9 Let $f:\mathbb{R}^2\to\mathbb{R}$ be defined by

$$f(x,y)=x^6-2x^2y-x^4y+2y^2.$$

($\mathbb{R}$ is the set of all real numbers and $\mathbb{R}^2=\{(x,y): x,y\in\mathbb{R}\}$)

Which one of the following statements is TRUE?

(A) f has a local maximum at origin

(B) f has a local minimum at origin

(C) f has a saddle point at origin

(D) The origin is not a critical point of f

Q.10 Let $\{a_n\}_{n=0}^{\infty}$ be any sequence of real numbers such that $\sum_{n=0}^{\infty} |a_n|^2 < \infty$. If the radius of convergence of $\sum_{n=0}^{\infty} a_n x^n$ is r, then which one of the following statements is necessarily TRUE?

(A) $r \geq 1$ or r is infinite

(B) $r < 1$

(C) $r = \left(\sum_{n=0}^{\infty} |a_n|^2 \right)^{\frac{1}{2}}$

(D) $r = \sum_{n=0}^{\infty} |a_n|^2$

Q.11 Let T_1 be the co-countable topology on $\mathbb{R}$ (the set of real numbers) and T_2 be the co-finite topology on $\mathbb{R}$.

Consider the following statements:

I. In $(\mathbb{R}, T_1)$, the sequence $\left\{ \frac{1}{n} \right\}_{n=1}^{\infty}$ converges to 0.

II. In $(\mathbb{R}, T_2)$, the sequence $\left\{ \frac{1}{n} \right\}_{n=1}^{\infty}$ converges to 0.

III. In $(\mathbb{R}, T_1)$, there is no sequence of rational numbers which converges to $\sqrt{3}$.

IV. In $(\mathbb{R}, T_2)$, there is no sequence of rational numbers which converges to $\sqrt{3}$.

Which of the above statements are TRUE?

(A) I and II only
(B) II and III only
(C) III and IV only
(D) I and IV only

Q.12 Let X and Y be normed linear spaces, and let $T : X \to Y$ be any bijective linear map with closed graph. Then which one of the following statements is TRUE?

(A) The graph of T is equal to $X \times Y$
(B) T^{-1} is continuous
(C) The graph of T^{-1} is closed
(D) T is continuous

Q.13 Let $g:\mathbb{R}^2 \to \mathbb{R}^2$ be a function defined by $g(x,y)=(e^x\cos y,\, e^x\sin y)$ and $(a,b)=g\left(1,\frac{\pi}{3}\right)$.

($\mathbb{R}$ is the set of all real numbers and $\mathbb{R}^2=\{(x,y): x,y\in\mathbb{R}\}$)

Which one of the following statements is TRUE?

(A) g is injective

(B) If h is the continuous inverse of g, defined in some neighbourhood of $(a,b)\in\mathbb{R}^2$, such that $h(a,b)=\left(1,\frac{\pi}{3}\right)$, then the Jacobian of h at (a,b) is e^2

(C) If h is the continuous inverse of g, defined in some neighbourhood of $(a,b)\in\mathbb{R}^2$, such that $h(a,b)=\left(1,\frac{\pi}{3}\right)$, then the Jacobian of h at (a,b) is e^{-2}

(D) g is surjective

Q.14 Let

$$u_n=\frac{n!}{1.3.5...(2n-1)},\ n\in\mathbb{N} \text{ (the set of all natural numbers).}$$

Then $\lim\limits_{n\to\infty} u_n$ is equal to _____ .

Q.15 If the differential equation

$$\frac{dy}{dx}=\sqrt{x^2+y^2},\ y(1)=2$$

is solved using the Euler's method with step-size $h=0.1$, then $y(1.2)$ is equal to _____ (round off to 2 places of decimal).

Q.16 Let f be any polynomial function of degree at most 2 over $\mathbb{R}$ (the set of all real numbers). If the constants a and b are such that

$$\frac{df}{dx}=a\,f(x)+2\,f(x+1)+b\,f(x+2),\ \text{for all } x\in\mathbb{R},$$

then $4a+3b$ is equal to _____ (round off to 2 places of decimal).

Q.17 Let L denote the value of the line integral $\oint_C (3x-4x^2y)dx+(4xy^2+2y)dy$, where C, a circle of radius 2 with centre at origin of the xy-plane, is traversed once in the anti-clockwise direction. Then $\frac{L}{\pi}$ is equal to _____ .

Q.18 The temperature $T:\mathbb{R}^3\setminus\{(0,0,0)\}\to\mathbb{R}$ at any point $P(x,y,z)$ is inversely proportional to the square of the distance of P from the origin. If the value of the temperature T at the point $R(0,0,1)$ is $\sqrt{3}$, then the rate of change of T at the point $Q(1,1,2)$ in the direction of $\overrightarrow{QR}$ is equal to _____ (round off to 2 places of decimal).

($\mathbb{R}$ is the set of all real numbers, $\mathbb{R}^3=\{(x,y,z):x,y,z\in\mathbb{R}\}$ and $\mathbb{R}^3\setminus\{(0,0,0)\}$ denotes $\mathbb{R}^3$ excluding the origin)

Q.19 Let f be a continuous function defined on $[0,2]$ such that $f(x)\geq 0$ for all $x\in[0,2]$. If the area bounded by $y=f(x)$, $x=0, y=0$ and $x=b$ is $\sqrt{3+b^2}-\sqrt{3}$, where $b\in(0,2]$, then $f(1)$ is equal to _____ (round off to 1 place of decimal).

Q.20 If the characteristic polynomial and minimal polynomial of a square matrix A are $(\lambda-1)(\lambda+1)^4(\lambda-2)^5$ and $(\lambda-1)(\lambda+1)(\lambda-2)$, respectively, then the rank of the matrix $A+I$ is _____, where I is the identity matrix of appropriate order.

Q.21 Let ω be a primitive complex cube root of unity and $i=\sqrt{-1}$. Then the degree of the field extension $\mathbb{Q}\left(i,\sqrt{3},\omega\right)$ over $\mathbb{Q}$ (the field of rational numbers) is _____.

Q.22 Let

$$\alpha=\int_C \frac{e^{i\pi z}\,dz}{2z^2-5z+2}, \quad C:\cos t+i\sin t,\ 0\le t\le 2\pi,\ i=\sqrt{-1}.$$

Then the greatest integer less than or equal to $|\alpha|$ is _____.

Q.23 Consider the system:

$$\begin{aligned} 3x_1+x_2+2x_3-x_4&=a,\\ x_1+x_2+x_3-2x_4&=3,\\ x_1,x_2,x_3,x_4&\ge 0. \end{aligned}$$

If $x_1=1,\ x_2=b,\ x_3=0,\ x_4=c$ is a basic feasible solution of the above system (where a, b and c are real constants), then $a+b+c$ is equal to _____.

Q.24 Let $f:\mathbb{C}\to\mathbb{C}$ be a function defined by $f(z)=z^6-5z^4+10$. Then the number of zeros of f in $\{z\in\mathbb{C}:|z|<2\}$ is _____.

($\mathbb{C}$ is the set of all complex numbers)

Q.25 Let

$$\ell^2=\{x=(x_1,x_2,...):x_i\in\mathbb{C},\ \sum_{i=1}^{\infty}|x_i|^2<\infty\}$$

be a normed linear space with the norm

$$\|x\|_2=\left(\sum_{i=1}^{\infty}|x_i|^2\right)^{\frac{1}{2}}.$$

Let $g:\ell^2\to\mathbb{C}$ be the bounded linear functional defined by

$$g(x)=\sum_{n=1}^{\infty}\frac{x_n}{3^n} \text{ for all } x=(x_1,x_2,...)\in\ell^2.$$

Then $\left(\sup\{|g(x)|:\|x\|_2\le 1\}\right)^2$ is equal to _____ (round off to 3 places of decimal).

($\mathbb{C}$ is the set of all complex numbers).

Q. 26 – Q. 55 carry two marks each.

Q.26 For the linear programming problem (LPP):

$$\begin{aligned} &\text{Maximize } Z = 2x_1 + 4x_2 \\ &\text{subject to } -x_1 + 2x_2 \le 4, \\ &\qquad 3x_1 + \beta x_2 \le 6, \\ &\qquad x_1, x_2 \ge 0, \ \beta \in \mathbb{R}, \end{aligned}$$

($\mathbb{R}$ is the set of all real numbers)

consider the following statements:

I. The LPP always has a finite optimal value for any $\beta \ge 0$.

II. The dual of the LPP may be infeasible for some $\beta \ge 0$.

III. If for some β, the point (1,2) is feasible to the dual of the LPP, then $Z \le 16$, for any feasible solution (x_1, x_2) of the LPP.

IV. If for some β, x_1 and x_2 are the basic variables in the optimal table of the LPP with $x_1 = \frac{1}{2}$, then the optimal value of dual of the LPP is 10.

Then which of the above statements are TRUE?

(A) I and III only
(B) I, III and IV only
(C) III and IV only
(D) II and IV only

Q.27 Let $f : \mathbb{R}^2 \to \mathbb{R}$ be defined by

$$f(x,y) = \begin{cases} (x^2+y^2)\sin\left(\dfrac{1}{x^2+y^2}\right), & \text{if } (x,y) \ne (0,0) \\ 0, & \text{if } (x,y) = (0,0). \end{cases}$$

Consider the following statements:

I. The partial derivatives $\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}$ exist at (0, 0) but are unbounded in any neighbourhood of (0, 0).

II. f is continuous but not differentiable at (0, 0).

III. f is not continuous at (0, 0).

IV. f is differentiable at (0, 0).

($\mathbb{R}$ is the set of all real numbers and $\mathbb{R}^2 = \{(x,y) : x, y \in \mathbb{R}\}$)

Which of the above statements is/are TRUE?

(A) I and II only
(B) I and IV only
(C) IV only
(D) III only

Q.28 Let $K=\left[k_{i,j}\right]_{i,j=1}^{\infty}$ be an infinite matrix over $\mathbb{C}$ (the set of all complex numbers) such that

(i) for each $i\in\mathbb{N}$ (the set of all natural numbers), the i^{th} row $\left(k_{i,1},k_{i,2},...\right)$ of K is in ℓ^{∞} and

(ii) for every $x=\left(x_1,x_2,...\right)\in\ell^1$, $\sum_{j=1}^{\infty}k_{i,j}\,x_j$ is summable for all $i\in\mathbb{N}$, and $(y_1,y_2,...)\in\ell^1$, where $y_i=\sum_{j=1}^{\infty}k_{i,j}\,x_j$.

Let the set of all rows of K be denoted by E. Consider the following statements:

P: E is a bounded set in ℓ^{∞}.

Q: E is a dense set in ℓ^{∞}.

$$\left(\ell^1=\{(x_1,x_2,...):x_i\in\mathbb{C},\ \sum_{i=1}^{\infty}|x_i|<\infty\}\right)$$
$$\left(\ell^{\infty}=\{(x_1,x_2,...):x_i\in\mathbb{C},\ \sup_{i\in\mathbb{N}}|x_i|<\infty\}\right)$$

Which of the above statements is/are TRUE?

(A) Both P and Q (B) P only (C) Q only (D) Neither P nor Q

Q.29 Consider the following heat conduction problem for a finite rod

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial x^2}-x e^t-2t,\quad t>0,\quad 0<x<\pi,$$

with the boundary conditions $u(0,t)=-t^2$, $u(\pi,t)=-\pi e^t-t^2$, $t>0$ and the initial condition $u(x,0)=\sin x-\sin^3 x-x$, $0\le x\le\pi$. If $v(x,t)=u(x,t)+x e^t+t^2$, then which one of the following is CORRECT?

(A) $v(x,t)=\frac{1}{4}\left(e^{-t}\ \sin x+e^{-9t}\ \sin 3x\right)$

(B) $v(x,t)=\frac{1}{4}\left(7e^{-t}\ \sin x-e^{-9t}\ \sin 3x\right)$

(C) $v(x,t)=\frac{1}{4}\left(e^{-t}\ \sin x+e^{-3t}\ \sin 3x\right)$

(D) $v(x,t)=\frac{1}{4}\left(3e^{-t}\ \sin x-e^{-3t}\ \sin 3x\right)$

Q.30 Let $f:\mathbb{C}\to\mathbb{C}$ be non-zero and analytic at all points in $\mathbb{Z}$.

If $F(z)=\pi f(z)\cot(\pi z)$ for $z\in\mathbb{C}\setminus\mathbb{Z}$, then the residue of F at $n\in\mathbb{Z}$ is _____ .

($\mathbb{C}$ is the set of all complex numbers, $\mathbb{Z}$ is the set of all integers and $\mathbb{C}\setminus\mathbb{Z}$ denotes the set of all complex numbers excluding integers)

(A) $\pi f(n)$ (B) $f(n)$ (C) $\dfrac{f(n)}{\pi}$ (D) $\left(\dfrac{df}{dz}\right)_{z=n}$

Q.31 Let the general integral of the partial differential equation

$$(2xy-1)\frac{\partial z}{\partial x}+\left(z-2x^2\right)\frac{\partial z}{\partial y}=2(x-yz)$$

be given by $F(u,v)=0$, where $F:\mathbb{R}^2\to\mathbb{R}$ is a continuously differentiable function.

($\mathbb{R}$ is the set of all real numbers and $\mathbb{R}^2=\{(x,y):x,y\in\mathbb{R}\}$)

Then which one of the following is TRUE?

(A) $u=x^2+y^2+z,\ v=xz+y$ (B) $u=x^2+y^2-z,\ v=xz-y$

(C) $u=x^2-y^2+z,\ v=yz+x$ (D) $u=x^2+y^2-z,\ v=yz-x$

Q.32 Consider the following statements:

I. If $\mathbb{Q}$ denotes the additive group of rational numbers and $f:\mathbb{Q}\to\mathbb{Q}$ is a non-trivial homomorphism, then f is an isomorphism.

II. Any quotient group of a cyclic group is cyclic.

III. If every subgroup of a group G is a normal subgroup, then G is abelian.

IV. Every group of order 33 is cyclic.

Which of the above statements are TRUE?

(A) II and IV only (B) II and III only

(C) I, II and IV only (D) I, III and IV only

Q.33 A solution of the Dirichlet problem

$$\nabla^2 u(r,\theta)=0,\quad 0<r<1,\ -\pi\le\theta\le\pi,$$
$$u(1,\theta)=|\theta|,\quad -\pi\le\theta\le\pi,$$

is given by

(A) $u(r,\theta)=\dfrac{\pi}{2}+\sum_{n=1}^{\infty}\left[\dfrac{(-1)^n-1}{n^2}\right]r^n\cos(n\theta)$

(B) $u(r,\theta)=\dfrac{2}{\pi}\sum_{n=1}^{\infty}\left[\dfrac{(-1)^{n+1}}{n^2}\right]r^n\cos(n\theta)$

(C) $u(r,\theta)=\dfrac{\pi}{2}+\dfrac{2}{\pi}\sum_{n=1}^{\infty}\left[\dfrac{(-1)^n-1}{n^2}\right]r^n\cos(n\theta)$

(D) $u(r,\theta)=\dfrac{\pi}{2}-\dfrac{2}{\pi}\sum_{n=1}^{\infty}\left[\dfrac{(-1)^n+1}{n^2}\right]r^n\cos(n\theta)$

Q.34 Consider the subspace $Y=\{(x,x):x\in\mathbb{C}\}$ of the normed linear space $\left(\mathbb{C}^2,\|\ \|_\infty\right)$.
If ϕ is a bounded linear functional on Y, defined by $\phi(x,x)=x$, then which one of the following sets is equal to

$$\left\{\psi(1,0):\psi \text{ is a norm preserving extension of } \phi \text{ to } \left(\mathbb{C}^2,\|\ \|_\infty\right)\right\}.$$

($\mathbb{C}$ is the set of all complex numbers, $\mathbb{C}^2=\{(x,y):x,y\in\mathbb{C}\}$ and
$\|(x_1,x_2)\|_\infty=\sup\{|x_1|,|x_2|\}$)

(A) $\{1\}$ (B) $\left[\dfrac{1}{2},\dfrac{3}{2}\right]$ (C) $[1,\infty)$ (D) $[0,1]$

Q.35 Consider the following statements:

I. The ring $\mathbb{Z}[\sqrt{-1}]$ is a unique factorization domain.

II. The ring $\mathbb{Z}[\sqrt{-5}]$ is a principal ideal domain.

III. In the polynomial ring $\mathbb{Z}_2[x]$, the ideal generated by x^3+x+1 is a maximal ideal.

IV. In the polynomial ring $\mathbb{Z}_3[x]$, the ideal generated by x^6+1 is a prime ideal.

($\mathbb{Z}$ denotes the set of all integers, $\mathbb{Z}_n$ denotes the set of all integers modulo n, for any positive integer n)

Which of the above statements are TRUE?

(A) I, II and III only (B) I and III only
(C) I, II and IV only (D) II and III only

Q.36 Let M be a 3×3 real symmetric matrix with eigenvalues 0, 2 and a with the respective eigenvectors $u=(4,b,c)^T$, $v=(-1,2,0)^T$ and $w=(1,1,1)^T$.

Consider the following statements:

I. $a+b-c=10$.

II. The vector $x=\left(0,\frac{3}{2},\frac{1}{2}\right)^T$ satisfies $Mx=v+w$.

III. For any $d\in\text{span}\{u,v,w\}$, $Mx=d$ has a solution.

IV. The trace of the matrix M^2+2M is 8.

(y^T denotes the transpose of the vector y)

Which of the above statements are TRUE?

(A) I, II and III only
(B) I and II only
(C) II and IV only
(D) III and IV only

Q.37 Consider the region

$$\Omega=\left\{x+i\,y:\ -1\le x\le 2,\ \frac{-\pi}{3}\le y\le\frac{\pi}{3}\right\},\ i=\sqrt{-1}$$

in the complex plane. The transformation $x+i\,y\mapsto e^{x+i\,y}$ maps the region Ω onto the region $S\subset\mathbb{C}$ (the set of all complex numbers). Then the area of the region S is equal to

(A) $\frac{\pi}{3}\left(e^4-e^{-2}\right)$
(B) $\frac{\pi}{4}\left(e^4+e^{-2}\right)$
(C) $\frac{2\pi}{3}\left(e^4-e^{-2}\right)$
(D) $\frac{\pi}{6}\left(e^4-e^{-2}\right)$

Q.38 Consider the sequence $\{g_n\}_{n=1}^{\infty}$ of functions, where $g_n(x)=\frac{x}{1+n\,x^2}$, $x\in\mathbb{R}$, $n\in\mathbb{N}$ and $g_n'(x)$ is the derivative of $g_n(x)$ with respect to x.

($\mathbb{R}$ is the set of all real numbers, $\mathbb{N}$ is the set of all natural numbers).

Then which one of the following statements is TRUE?

(A) $\{g_n\}_{n=1}^{\infty}$ does **NOT** converge uniformly on $\mathbb{R}$

(B) $\{g_n'\}_{n=1}^{\infty}$ converges uniformly on any closed interval which does **NOT** contain 1

(C) $\{g_n'\}_{n=1}^{\infty}$ converges point-wise to a continuous function on $\mathbb{R}$

(D) $\{g_n'\}_{n=1}^{\infty}$ converges uniformly on any closed interval which does **NOT** contain 0

Q.39 Consider the boundary value problem (BVP)

$$\frac{d^2y}{dx^2} + \alpha\, y(x) = 0,\ \alpha \in \mathbb{R} \text{ (the set of all real numbers)},$$

with the boundary conditions $y(0)=0, y(\pi)=k$ (k is a non-zero real number).

Then which one of the following statements is TRUE?

(A) For $\alpha = 1$, the BVP has infinitely many solutions

(B) For $\alpha = 1$, the BVP has a unique solution

(C) For $\alpha = -1, k < 0$, the BVP has a solution $y(x)$ such that $y(x) > 0$ for all $x \in (0,\pi)$

(D) For $\alpha = -1, k > 0$, the BVP has a solution $y(x)$ such that $y(x) > 0$ for all $x \in (0,\pi)$

Q.40 Consider the ordered square I_0^2, the set $[0,1]\times[0,1]$ with the dictionary order topology. Let the general element of I_0^2 be denoted by $x \times y$, where $x, y \in [0,1]$. Then the closure of the subset

$$S = \left\{ x \times \frac{3}{4} : 0 < a < x < b < 1 \right\} \text{ in } I_0^2$$

is

(A) $S \cup \left((a,b]\times\{0\}\right) \cup \left([a,b)\times\{1\}\right)$

(B) $S \cup \left([a,b)\times\{0\}\right) \cup \left((a,b]\times\{1\}\right)$

(C) $S \cup \left((a,b)\times\{0\}\right) \cup \left((a,b)\times\{1\}\right)$

(D) $S \cup \left((a,b]\times\{0\}\right)$

Q.41 Let P_2 be the vector space of all polynomials of degree at most 2 over $\mathbb{R}$ (the set of real numbers). Let a linear transformation $T : P_2 \to P_2$ be defined by

$$T\left(a + bx + cx^2\right) = (a+b) + (b-c)\,x + (a+c)\,x^2.$$

Consider the following statements:

I. The null space of T is $\left\{\alpha\left(-1 + x + x^2\right) : \alpha \in \mathbb{R}\right\}$.

II. The range space of T is spanned by the set $\left\{1 + x^2, 1 + x\right\}$.

III. $T(T(1+x)) = 1 + x^2$.

IV. If M is the matrix representation of T with respect to the standard basis $\left\{1, x, x^2\right\}$ of P_2, then the trace of the matrix M is 3.

Which of the above statements are TRUE?

(A) I and II only

(B) I, III and IV only

(C) I, II and IV only

(D) II and IV only

Q.42 Let T_1 and T_2 be two topologies defined on $\mathbb{N}$ (the set of all natural numbers), where T_1 is the topology generated by $\mathcal{B} = \{\{2n-1, 2n\} : n \in \mathbb{N}\}$ and T_2 is the discrete topology on $\mathbb{N}$.

Consider the following statements :

I. In $(\mathbb{N}, T_1)$, every infinite subset has a limit point.

II. The function $f : (\mathbb{N}, T_1) \to (\mathbb{N}, T_2)$ defined by

$$f(n) = \begin{cases} \frac{n}{2}, & \text{if } n \text{ is even} \\ \frac{n+1}{2}, & \text{if } n \text{ is odd} \end{cases}$$

is a continuous function.

Which of the above statements is/are TRUE?

(A) Both I and II (B) I only

(C) II only (D) Neither I nor II

Q.43 Let $1 \le p < q < \infty$. Consider the following statements:

I. $\ell^p \subset \ell^q$

II. $L^p[0,1] \subset L^q[0,1]$,

where $\ell^p = \{(x_1, x_2, \ldots) : x_i \in \mathbb{R}, \sum_{i=1}^{\infty} |x_i|^p < \infty\}$ and

$$L^p[0,1] = \left\{ f : [0,1] \to \mathbb{R} : f \text{ is } \mu\text{-measurable}, \int_{[0,1]} |f|^p \, d\mu < \infty, \text{where } \mu \text{ is the Lebesgue measure} \right\}$$

($\mathbb{R}$ is the set of all real numbers)

Which of the above statements is/are TRUE?

(A) Both I and II (B) I only (C) II only (D) Neither I nor II

Q.44 Consider the differential equation

$$t\frac{d^2y}{dt^2}+2\frac{dy}{dt}+t\,y=0,\ t>0,\ y(0+)=1,\ \left(\frac{dy}{dt}\right)_{t=0+}=0.$$

If $Y(s)$ is the Laplace transform of $y(t)$, then the value of $Y(1)$ is _____ (round off to 2 places of decimal).

(Here, the inverse trigonometric functions assume principal values only)

Q.45 Let R be the region in the xy-plane bounded by the curves $y=x^2,\ y=4x^2,\ xy=1$ and $xy=5$.

Then the value of the integral $\iint_R \frac{y^2}{x}\,dy\,dx$ is equal to _____.

Q.46 Let V be the vector space of all 3×3 matrices with complex entries over the real field. If

$$W_1=\{A\in V : A=\bar{A}^T\} \quad \text{and} \quad W_2=\{A\in V : \text{trace of } A=0\},$$

then the dimension of W_1+W_2 is equal to _____.

($\bar{A}^T$ denotes the conjugate transpose of A)

Q.47 The number of elements of order 15 in the additive group $\mathbb{Z}_{60}\times\mathbb{Z}_{50}$ is _____ .

($\mathbb{Z}_n$ denotes the group of integers modulo n, under the operation of addition modulo n, for any positive integer n)

Q.48 Consider the following cost matrix of assigning four jobs to four persons:

Jobs

Persons	J_1	J_2	J_3	J_4
P_1	5	8	6	10
P_2	2	5	4	8
P_3	6	7	6	9
P_4	6	9	8	10

Then the minimum cost of the assignment problem subject to the constraint that job J_4 is assigned to person P_2, is _____ .

Q.49 Let $y:[-1,1]\to\mathbb{R}$ with $y(1)=1$ satisfy the Legendre differential equation

$$\left(1-x^2\right)\frac{d^2y}{dx^2}-2x\frac{dy}{dx}+6y=0 \text{ for } |x|<1.$$

Then the value of $\int_{-1}^{1} y(x)\left(x+x^2\right)dx$ is equal to _____ (round off to 2 places of decimal).

Q.50 Let $\mathbb{Z}_{125}$ be the ring of integers modulo 125 under the operations of addition modulo 125 and multiplication modulo 125. If m is the number of maximal ideals of $\mathbb{Z}_{125}$ and n is the number of non-units of $\mathbb{Z}_{125}$, then $m+n$ is equal to _____ .

Q.51 The maximum value of the error term of the composite Trapezoidal rule when it is used to evaluate the definite integral

$$\int_{0.2}^{1.4}\left(\sin x-\log_e x\right)dx$$

with 12 sub-intervals of equal length, is equal to _____ (round off to 3 places of decimal).

Q.52 By the Simplex method, the optimal table of the linear programming problem:

$$\begin{aligned}\text{Maximize } & Z=\alpha x_1+3x_2\\ \text{subject to } & \beta x_1+x_2+x_3=8,\\ & 2x_1+x_2+x_4=\gamma,\\ & x_1, x_2, x_3, x_4\geq 0,\end{aligned}$$

where α,β,γ are real constants, is

$c_j \rightarrow$	α	3	0	0	
Basic variable	x_1	x_2	x_3	x_4	Solution
x_2	1	0	2	-1	6
x_1	0	1	-1	1	2
z_j-c_j	0	0	2	1	-

Then the value of $\alpha+\beta+\gamma$ is _____ .

Q.53 Consider the inner product space P_2 of all polynomials of degree at most 2 over the field of real numbers with the inner product $\langle f, g \rangle = \int_0^1 f(t)\, g(t)\, dt$ for $f, g \in P_2$.

Let $\{f_0, f_1, f_2\}$ be an orthogonal set in P_2, where $f_0 = 1, f_1 = t + c_1, f_2 = t^2 + c_2\, f_1 + c_3$ and c_1, c_2, c_3 are real constants. Then the value of $2c_1 + c_2 + 3c_3$ is equal to _____ .

Q.54 Consider the system of linear differential equations

$$\frac{dx_1}{dt} = 5x_1 - 2x_2,$$

$$\frac{dx_2}{dt} = 4x_1 - x_2,$$

with the initial conditions $x_1(0) = 0,\ x_2(0) = 1$.

Then $\log_e\left(x_2(2) - x_1(2)\right)$ is equal to _____ .

Q.55 Consider the differential equation

$$x(1+x^2)\frac{d^2y}{dx^2} - 9\frac{dy}{dx} + 7y = 0.$$

The sum of the roots of the indicial equation of the Frobenius series solution for the above differential equation in a neighborhood of $x = 0$ is equal to _____.

END OF THE QUESTION PAPER

Q.No.	Type	Section	Key	Marks
1	MCQ	GA	C	1
2	MCQ	GA	C	1
3	MCQ	GA	C	1
4	MCQ	GA	C	1
5	MCQ	GA	A	1
6	MCQ	GA	C	2
7	MCQ	GA	A	2
8	MCQ	GA	C	2
9	MCQ	GA	B	2
10	MCQ	GA	D	2
1	MCQ	MA	D	1
2	MCQ	MA	A	1
3	MCQ	MA	D	1
4	MCQ	MA	D	1
5	MCQ	MA	C	1
6	MCQ	MA	C	1
7	MCQ	MA	D	1
8	MCQ	MA	B	1
9	MCQ	MA	C	1
10	MCQ	MA	A	1
11	MCQ	MA	B	1
12	MCQ	MA	C	1
13	MCQ	MA	C	1

Q.No.	Type	Section	Key	Marks
14	NAT	MA	0 to 0	1
15	NAT	MA	2.40 to 2.50	1
16	NAT	MA	-7.5 to -7.5	1
17	NAT	MA	31.90 to 32.10	1
18	NAT	MA	0.21 to 0.23	1
19	NAT	MA	0.5 to 0.5	1
20	NAT	MA	6 to 6	1
21	NAT	MA	4 to 4	1
22	NAT	MA	2 to 2	1
23	NAT	MA	7 to 7	1
24	NAT	MA	4 to 4	1
25	NAT	MA	0.125 to 0.125	1
26	MCQ	MA	B	2
27	MCQ	MA	B	2
28	MCQ	MA	B	2
29	MCQ	MA	A	2
30	MCQ	MA	B	2
31	MCQ	MA	A	2
32	MCQ	MA	C	2
33	MCQ	MA	C	2
34	MCQ	MA	D	2
35	MCQ	MA	B	2
36	MCQ	MA	B	2

Q.No.	Type	Section	Key	Marks
37	MCQ	MA	A	2
38	MCQ	MA	D	2
39	MCQ	MA	D	2
40	MCQ	MA	A	2
41	MCQ	MA	C	2
42	MCQ	MA	A	2
43	MCQ	MA	B	2
44	NAT	MA	0.76 to 0.83	2
45	NAT	MA	12 to 12	2
46	NAT	MA	17 to 17	2
47	NAT	MA	48 to 48	2
48	NAT	MA	27 to 27	2
49	NAT	MA	0.25 to 0.30	2
50	NAT	MA	26 to 26	2
51	NAT	MA	0.022 to 0.028	2
52	NAT	MA	14.0 to 16.0 OR 17.0 to 18.5	2
53	NAT	MA	-3 to -3	2
54	NAT	MA	1.95 to 2.05	2
55	NAT	MA	10 to 10	2